Exercise series n°3 - Semester 1
2025-2026
Binary relations

Activity 1:
We define the relation $\mathcal{R}$ on $\mathbb{R}^*$ by

$$x\mathcal{R}y \text{ iff } x^2 - \frac{1}{x^2} = y^2 - \frac{1}{y^2}.$$

1. Show that $\mathcal{R}$ is an equivalence relation.
2. Determine the equivalence class of $a \in \mathbb{R}^*$

Activity 2:
On $\mathbb{R}$, we define the relation $\Re$ by setting $x \,\Re\, y \quad \text{iff} \quad x^2 - y^2 = x - y$.

1. Prove that $\Re$ is an equivalence relation.
2. Compute the equivalence class of an element x of $\mathbb{R}$. How many elements are there in this class?

Activity 3:
Let E denotes a given set. We consider the set $P(E)$ of subsets of E, We define the folowing relation:

$$A \,\Re\, B \;\text{ iff }\; A = B \;\textit{ or }\; A = \bar{B}$$

Prove that $\Re$ is an equivalence relation.

Activity 4:
Let $\preceq$ be the binary relation defined in $\mathbb{N}^*$ by :

$$p \preceq q \text{ iff } \exists \; n \in \mathbb{N}^* \; t.q \;\; p^n = q.$$

1. Prove that $\preceq$ is an ordering relation.
2. Is this order total or partial?
3. Prove that $\{2, 3\}$ is upper bounded (has a greatest element) in $(\mathbb{N}^*, \preceq)$.

Activity 5:

1. Prove that the relation $\mathcal{R}'$ defined on $\mathbb{R}$ by:
$$x\mathcal{R}'y \Longleftrightarrow xe^y = ye^x$$
is an equivalence relation.
2. Precise, for x fixed in $\mathbb{R}$, the class's elements number of x modulo $\mathcal{R}$.

Activity 6:
On $\mathbb{R}^2$, we consider the relation defined by:

$$(a, b)\pounds(c, d) \text{ iff } \; a^2 + b^2 = c^2 + d^2.$$

1. Prove that $\pounds$ is an equivalence relation.
2. Describe the equivalence class (a, b) of the pair (a, b).
3. Let $\mathbb{R}^2_{/\pounds}$ be the quotient set for this relation. Prove that the application
$$\begin{aligned} \mathbb{R}^2_{/\pounds} &\to [0, +\infty[\\ (a, b) &\mapsto a^2 + b^2 \end{aligned}$$
is well-defined and that it is a bijection.